

# Information encoding in homoclinic chaotic systems

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In this work we describe a simple method of encoding information at real time in the inter-spike intervals of a homoclinic chaotic system. This has been experimentally tested by means of an instantaneous synchronization between the laser intensity of a CO<sub>2</sub> laser with feedback in the regime of Sil'nikov chaos and an external pulsed signal of very low power. The information is previously encoded in the temporal intervals between consecutive pulses of the external signal. The value of the inter-pulse intervals is varied each time a new pulse is generated.

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Chaotic carriers are exploited to encode and transmit signals via two distinct approaches. The first method [1] consists in applying control techniques [2] in order to stabilize one of the uncountably infinite chaotic orbits embedded within the chaotic attractor, such that its intersections with the Poincaré section can be mapped into a desired sequence of bits [3]. The second method, that has been more extensively used, consists in exploiting the synchronization properties of coupled chaotic systems [4] in order to modulate a chaotic carrier with a message signal at the transmitter, and then recover the message by demodulating the synchronized state at the receiver. In the past, the latter method has been mainly focussed on the problem of warranting privacy in the communication [5], but it has recently found applications for communication with chaotic time-delayed optical systems [6].

In this Letter we discuss an alternative procedure which makes use of pulse synchronization to encode desired messages into the interspike interval sequences of a homoclinic chaotic system. This way, the information is coded only in the time intervals at which spikes occur, and does not affect any geometrical property of the chaotic flow, thus resulting in a better performance against unwanted perturbations, as e.g. noise contamination in the communication channel. We will discuss the performance of the proposed method in an experiment on a CO<sub>2</sub> laser displaying a regime of Sil'nikov chaos.

Sil'nikov chaos [7] has been observed in many systems, such as chemical [8] and laser [9] experiments. This kind of behavior shows striking similarities with the electrical spike trains travelling on the axons of animal neurons [10, 11]. More generally, chemical oscillators based on an activator-inhibitor competition, which rule biological clocks controlling living rhythms, such as the heart pacemaker, hormone production, metabolism, etc., display rhythmical trains of spikes with erratic repetition frequency as shown in Refs. [8, 9].

Homoclinic chaos of the Sil'nikov type normally appears when a parameter is varied towards the homoclinic condition associated with a saddle focus [7, 12, 13]. Its

peculiarity consists of an astonishing regularity of the geometric trajectory in phase space. The chaotic motion is characterized by the large fluctuations in the return time of consecutive spikes, so that only an average return period can be defined. Thus, an appropriate indicator of chaos may be the distribution of the return times to a given threshold, and the strength of chaos is associated with the amount of decorrelation between successive returns [14]. Such decorrelation occurs around the saddle focus, when the system displays a large susceptibility, i.e. a large response to an external perturbation [15]; we will exploit this susceptibility for information encoding.

Recently, different phase locking regimes between an initial homoclinic chaotic signal and a sinusoidal periodic external modulation has been reported [16]. The initial homoclinic chaotic behavior can be suppressed by the action of a periodic external forcing, giving rise to different phase locking domains, (1:1, 1:2, 1:3 and 2:1), depending on the frequency of the applied periodic forcing. The applied forcing readjusts the evolution of the dynamics of the homoclinic chaotic signal in such a way that the return period of every orbit in phase space is the same. Thus, a periodic orbit is stabilized.

Based on this synchronization, we propose an innovative and viable method of controlling instantaneously the temporal interval between spikes. In this way, the return period of each cycle changes accordingly to an external information. In order to get an exact control in the return period of each orbit, it is fundamental to minimize the response time of the system to the forcing signal. For this purpose, the external perturbation is made of short pulses. In this way, synchronization between both signals is obtained almost instantaneously, thus achieving a robust control on the individual inter-spike interval of a homoclinic chaotic system, and hence encoding the desired information in the sequence of inter-spikes.

The experiment has been performed on a single mode CO<sub>2</sub> laser with a feedback proportional to the output intensity (see Ref. [16]). The control parameters, bias voltage and gain of the feedback, are set in a regime of

homoclinic chaotic behavior (see Fig. 1(a)). Figure 1(b) shows the phase space reconstruction of the chaotic attractor. The modulating external signal is applied, in an additive way, on the bias parameter with a waveform generator Tektronix TM5003, which is controlled by a real time PC board (PCI-7030/6040E) from National Instruments. The perturbation signal is a train of square waveforms. The inter-pulse intervals can be controlled, in real time, by means of an adequate computer program. Each time a new pulse is generated, this distance in time is changed, to encode in these time intervals the desired information. The inter-pulse interval is varied inside the domain of 1:1 phase locking for the external perturbation and the laser intensity. This time variation range corresponds to a frequency of repetition of pulses that can take values in a quite large interval. The time duration of each pulse is 10% of the inter-pulse interval. The amplitude of the pulses is 4% of the value of the amplitude of the main spikes of the temporal evolution of the laser intensity. The laser output is recorded by a digital oscilloscope with a sampling time of  $1 \mu s$ .

The first performance index we have studied is the correlation between the different time (or frequency) intervals of consecutive pulses of the external signal and the corresponding return periods (or frequencies) in the temporal series of the laser intensity. This index provides information on the amount of synchronization between the external forcing and the signal to control, initially in a homoclinic chaotic regime. Performance has been evaluated for a random uniform frequency distribution of the forcing signal centered in the middle value of the corresponding Arnold tongue associated with the 1:1 phase locking, that is, centered at  $f_0 = 1.7 KHz$  (see Ref. [16]). Thus, the frequency of the external modulation take values in the interval  $(f_0 - f_s, f_0 + f_s)$  provided that the 1:1 phase locking is guaranteed, that is, for  $f_s \leq 0.6 KHz$ .

As shown in Fig. 2(a), the phase of the temporal series of the laser intensity follows almost perfectly the external signal, with the laser spikes adjusting in each orbit to the corresponding external signal. This is obtained because the laser parameters are adjusted to the homoclinic chaos regime where the system crosses a region of large susceptibility [15]. Figure 2(b) shows the inter-spike intervals of the laser intensity versus the inter-pulse intervals of the modulating signal. Synchronization is expressed by the straight line of 45 degrees. It is interesting to notice that synchronization does not depend on the frequency range of the external modulation. A better estimation of the error in time between the inter-spike intervals of the laser and the timing between pulses of the external modulating signal is shown in Fig. 2(c), where an histogram of these differences in time is plotted. Such histogram is fitted by a zero-mean Gaussian distribution with a standard deviation of  $7.8 \mu s$ . In this figure we also appreciate that the difference between the corresponding return periods of the external signal and the laser intensity is always

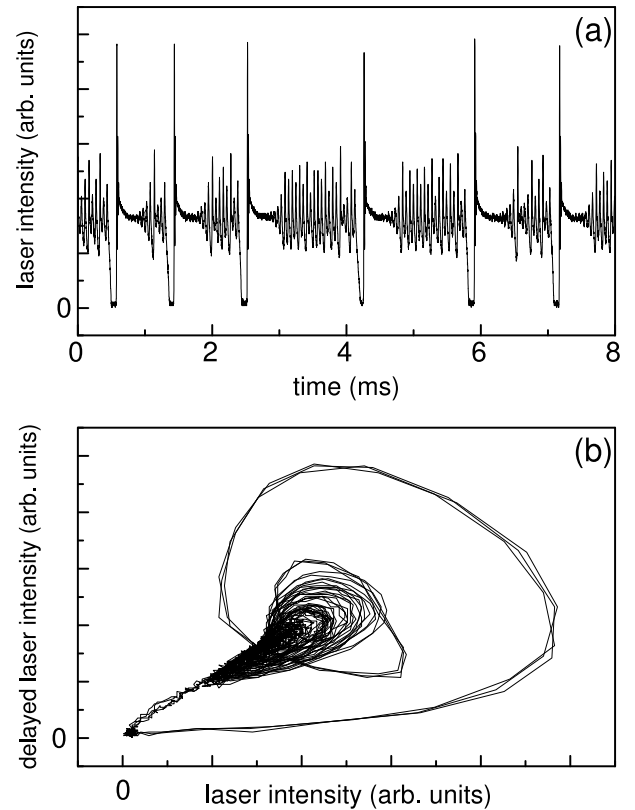


FIG. 1: (a) Experimental time series of the laser intensity for a  $CO_2$  laser with feedback, in the regime of homoclinic chaos. The intensity modulation has been amplified and lifted for visual convenience. Homoclinic chaotic behavior in a  $CO_2$  laser. (b) Phase space trajectory built by an embedding technique with appropriate delay.

below a small value ( $\Delta t \leq 25 \mu s$ ).

From this analysis, we conclude that it is possible to control the laser with pulses of very low power. Thus, it is possible to encode in the return periods of the laser intensity messages of binary, ternary, quaternary nature, etc... The fact of encoding information in the timing between spikes can be very useful in order to design a robust communication system [17] or to achieve a better understanding in the communication phenomenon among cells in the central nervous system [18]. It is well known that the pattern of spike times provides a large capacity for conveying information beyond that due to the code commonly assumed by physiologists, the number of spikes fired [19].

As an example of how to encode information in the timing between spikes, we present the simple case in which a binary message is encoded in the return period of the spikes in the  $CO_2$  laser intensity. The homoclinic chaotic behavior of the laser can be used to encode a desired message in such a way that the bits "1" and "0" are easily identified as timing between spikes less or greater than a threshold value. The encoding procedure is the follow-

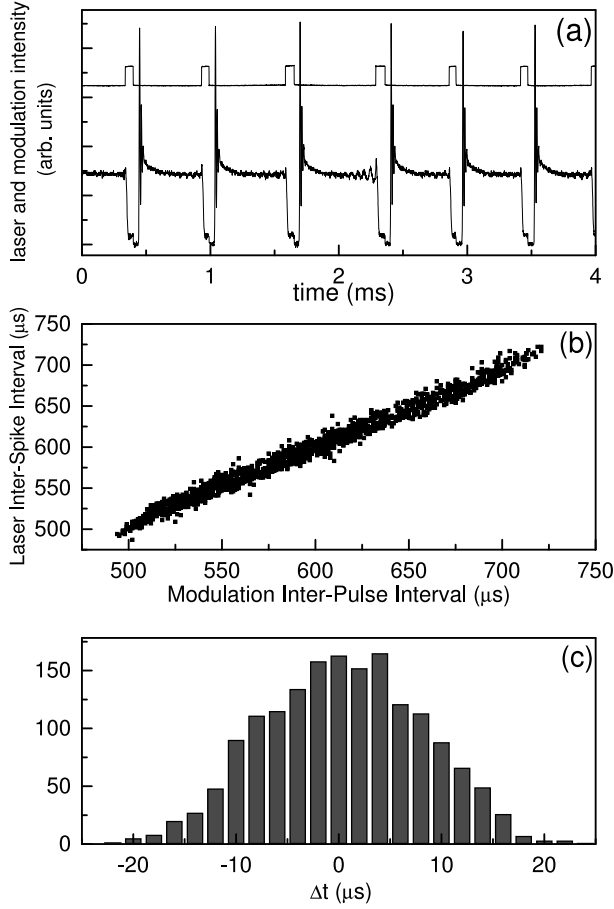


FIG. 2: (a) Experimental temporal evolution of the laser intensity driven by an external pulsed signal. (b) Inter-spike intervals of the laser intensity versus inter-pulse intervals of the forcing signal. This plot indicates synchronization between the two variables. (c) Histogram of the time differences ( $\Delta t$ ) between inter-pulse intervals of the external signal and inter-spike intervals of the laser intensity.

ing. First of all, an external pulsed signal that modulates the laser intensity is synthesized with an inter-pulse frequency changed after each pulse according to a uniform frequency distribution centered at the mean repetition frequency,  $f_0 = 1.7 \text{ KHz}$ . When the value of the frequency is larger than  $f_0$ , a bit '1' is encoded in the external signal; when this value is lower than  $f_0$ , a bit '0' is encoded in the external signal. Since the error in time in the synchronization process between the external signal and the laser intensity is below  $25 \mu\text{s}$ , in order to avoid errors around the mean frequency, we consider variations in the frequency in the interval  $(f_0 - f_s, f_0 - f_a)$  to encode the bit '0', and in the interval  $(f_0 + f_a, f_0 + f_s)$  to encode the bit '1'. It is obvious that for  $f_a \geq 0.08 \text{ KHz}$  no errors in the transmission are guaranteed, since this is the variation in the frequency associated with a time variation of  $25 \mu\text{s}$  ( $\Delta f = \Delta t * f_0^2$ ). Figure 3(a) shows the histogram of the inter-spike intervals of the free-running

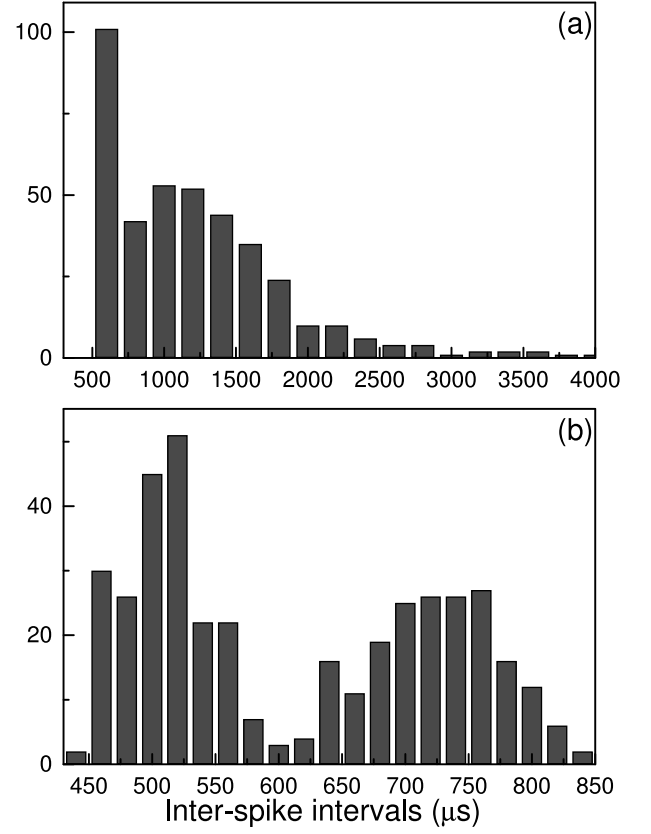


FIG. 3: a) Histogram of the frequencies of the free-running signal before applying any perturbation. (b) Histogram when some information has been encoded in the signal. The information has been considered as a uniform distribution of bits '1' and bits '0'.

signal before applying any perturbation, and Figure 3(b) shows the same when some information has been encoded in the own signal. The information has been considered as a uniform distribution of bits '1' and bits '0'. We can appreciate in these histograms how a left shift in time (right shift in frequency) is produced when the control is applied. This is easy to understand since the perturbation tends to eliminate the oscillations of the laser intensity around the saddle point, leading such variable to the rest state (no intensity). In this way, the orbit is shortened and the time difference between consecutive spikes is reduced. Notice also that the frequency range of the modulating signal to achieve 1:1 phase locking is centered around the most probable value of the frequency corresponding to the homoclinic behavior of the laser intensity.

The information decoding process in the laser intensity can be carried out by observing the instant when the laser intensity is higher than a threshold level. The fact that the spike amplitude is very high in comparison with the signal elsewhere, and the fact that the system can be controlled with small perturbations of very low power,

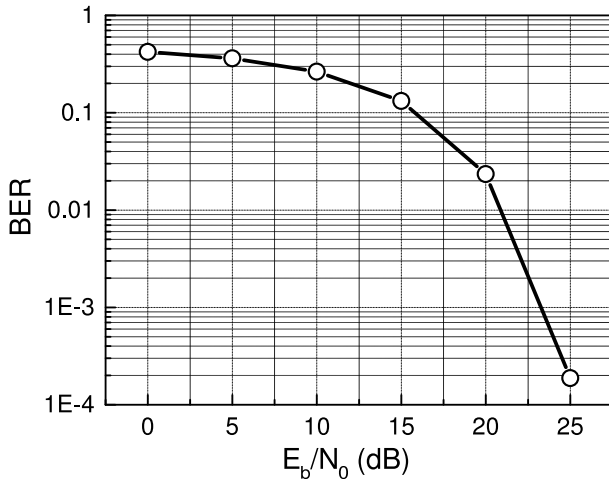


FIG. 4: BER vs.  $E_b/N_0$  of the proposed communication system when additive white Gaussian noise is contaminating the channel.

make it suitable for information transmission in a communication system. Since the information is contained entirely in the timing between spikes, channel distortions that affect the pulse shape will not significantly influence the information transmission. From this point of view, we analyze how robust the system is to the addition of a zero-mean Additive White Gaussian Noise (AWGN) in the communication channel. Notice that the channel noise is added to the transmitted signal as  $s(t) + n(t)$  in the transmission channel and sent to the receiver. Figure 4 shows the Bit Error Rate (BER) versus  $E_b/N_0$ , where  $E_b$  is the energy per bit, and  $N_0/2$  represents the power spectral density of the channel noise. We can see in this figure how the BER strongly decreases as the  $E_b/N_0$  relation increases. For values of  $E_b/N_0 \geq 25$  dB the BER is already below  $10^{-3}$ . Our method provides a BER consistently lower than other recent proposals [20].

In conclusion, we have shown experimentally how it is possible to control the inter-spike intervals of a homoclinic chaotic system by applying small perturbations. This fact allows to encode information in the timing between spikes. This can be useful in order to achieve a better understanding in the communication phenomenon among cells in the central nervous system as well as to design a robust communication system. In particular, we have shown a simple method to encode a binary message in a homoclinic chaotic signal that can be used in communication applications.

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